Macroscopic GHZ-correlations

In

Superconducting Josephson systems

PRL 96, 246803 (2006) PHYSICAL REVIEW LETTERS week ending 23 JUNE 2006

Generation and Control of Greenberger-Horne-Zeilinger Entanglement in Superconducting Circuits

L. F. Wei,[1,2] Yu-xi Liu,[1] and Franco Nori[1,3]

无言独上西楼，月如钩，寂寞梧桐深院锁清秋。

剪不断，理还乱，是离愁，别是一般滋味在心头。

【南唐】李煜《相见欢》

What is entanglement?

Schrödinger`s Cat(1935):

Einstein et al`s description(1935):

$$\left|\mathrm{EPR}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow_1\downarrow_2\right\rangle - \left|\downarrow_1\uparrow_2\right\rangle\right)$$

$$\left|\text{S-Cat}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\text{excited, alive}\right\rangle + \left|\text{decayed, dead}\right\rangle\right)$$

Greenberger

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle|0\rangle + |1\rangle|1\rangle|1\rangle\right)$$

Horne

Zeilinger

GOING BEYOND BELL'S THEOREM

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[2]*Stonehill College, North Easton, Massachusetts*
[3]*Atominstitut der Oesterreichischen Universitaeten, Wien, Austria*

In Bell's inequality, Quantum theory, and onceptions of the universe, P73-76, 1989

PHYSICAL REVIEW LETTERS

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Observation of Three-Photon Greenberger-Horne-Zeilinger Entanglement

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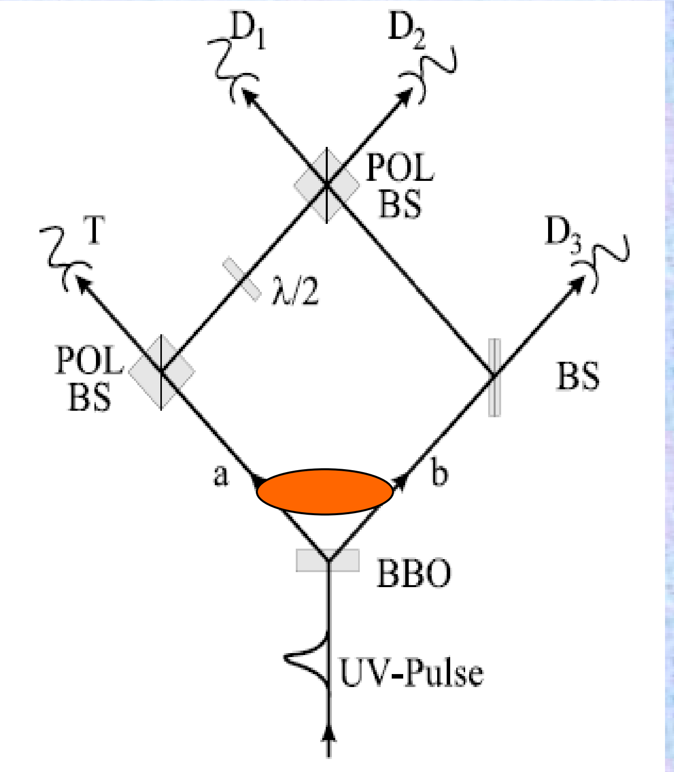
(Received 6 October 1998)



$$\frac{1}{\sqrt{2}}|H\rangle_T(|H\rangle_1|H\rangle_2|V\rangle_3 + |V\rangle_1|V\rangle_2|H\rangle_3)$$

Probably

$$\frac{1}{2}\{|H\rangle_T(|H\rangle'_1|H\rangle'_2|V\rangle_3 + |V\rangle'_1|V\rangle_2|H\rangle'_3) + |H\rangle'_T(|H\rangle_1|H\rangle_2|V\rangle'_3 + |V\rangle_1|V\rangle'_2|H\rangle_3)\}$$

$$|H\rangle_a \rightarrow |H\rangle_T\,,$$

$$|V\rangle_b \rightarrow \frac{1}{\sqrt{2}}(|V\rangle_2 + |V\rangle_3)$$

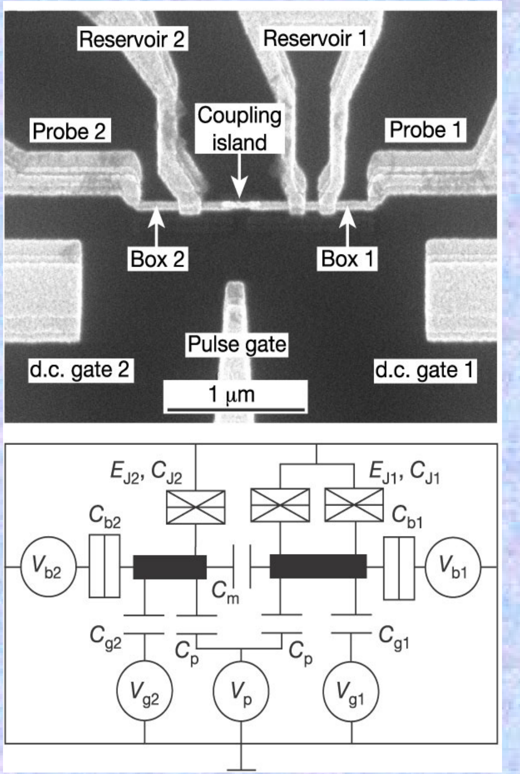
$$|V\rangle_a \rightarrow \frac{1}{\sqrt{2}}(|V\rangle_1 + |H\rangle_2)$$

$$|H\rangle_b \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_1 + |H\rangle_3)$$

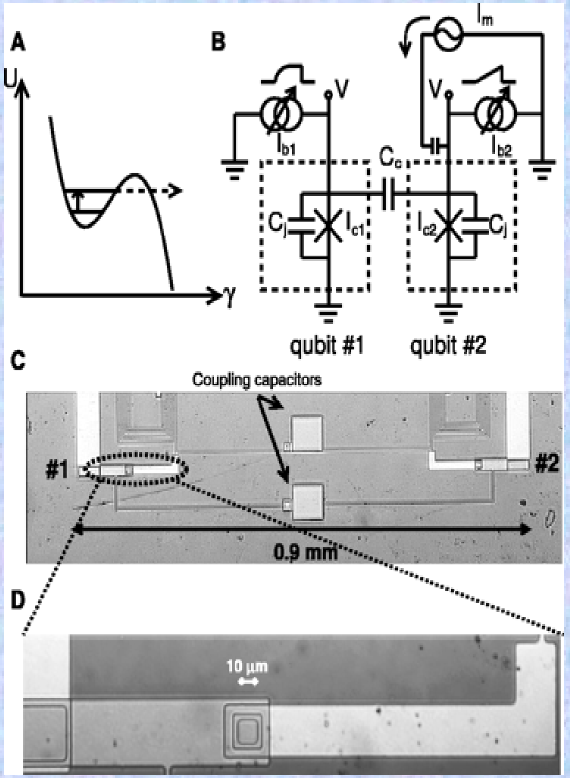
$$\frac{1}{2}(|H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b)(|H\rangle'_a|V\rangle'_b - |V\rangle'_a|H\rangle'_b)$$

How to generate a macroscopic GHZ-state?

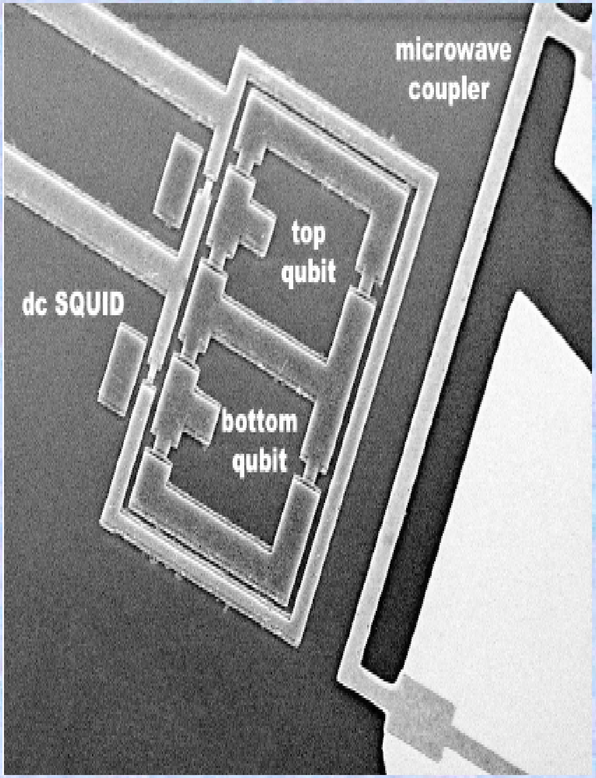
---Experimentally constantly-coupled Josephson circuits



Yu. A. PASHKIN et al, Nature 421, 823 (2003)



Berkley et al,* *Science, 300, 1548 (2003)

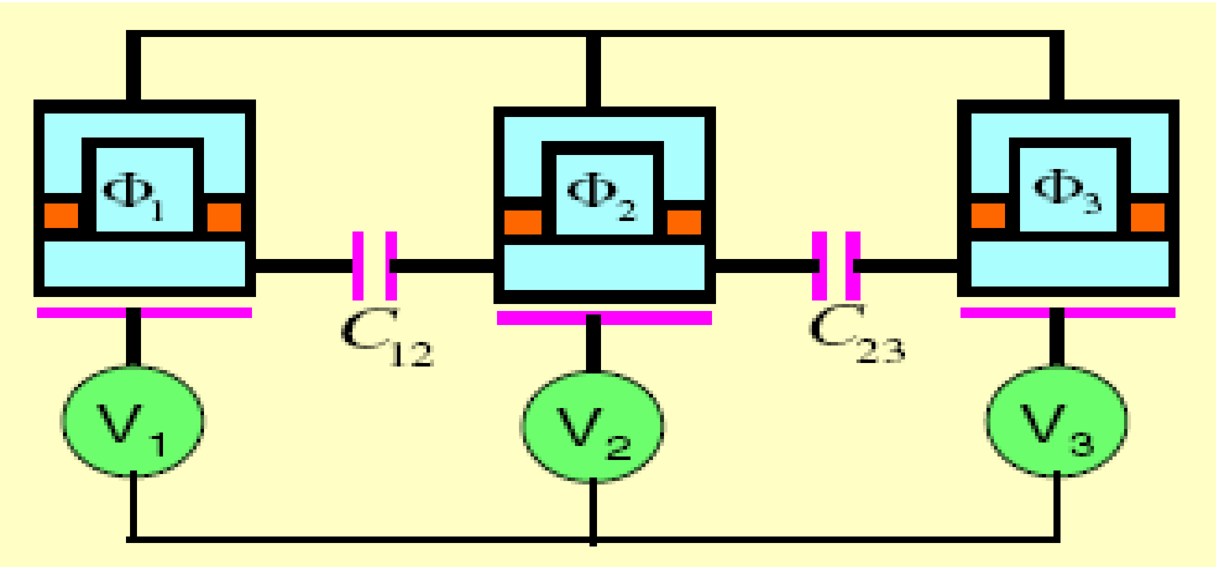


J. B. Majer ***et al,*** Phys. Rev. Lett. **94**, 090501 (2005)

PRL 96, 246803 (2006) PHYSICAL REVIEW LETTERS week ending 23 JUNE 2006

Generation and Control of Greenberger-Horne-Zeilinger Entanglement in Superconducting Circuits

L. F. Wei,[1,2] Yu-xi Liu,[1] and Franco Nori[1,3]



$$\hat{H} = \frac{1}{2}\sum_{j=1}^{3}[E_{C_j}(\{V_j\})\sigma_j^z - E_{J_j}(\Phi_j)\sigma_j^x] + \sum_{j=1}^{2} K_{j,j+1}\sigma_j^z\sigma_{j+1}^z$$

with

$$K_{j,j+1} = e^2/\tilde{C}_{j,j+1}$$

1. Three-step scheme for generating GHZ-state

$$|000\rangle \xrightarrow[\cos(E_{J_2}t_1/2\hbar)=1/\sqrt{2}]{E_{J_2}=E_{J_3}=0,\ \delta E_{C_1}=E_{12}} \frac{1}{\sqrt{2}}(|000\rangle + i|100\rangle)$$

to keep the state $|000\rangle$ unchange, we need the length of pulse further satisfies the condiditon $\sin(t_2\sqrt{(E_{J_2}/2)^2+E_{12}^2}/\hbar)=0$ by setting E_{J_2} properly.

$E_{J_1}=E_{J_3}=0,\ \cos(E_{J_2}t_2/2\hbar)=0,\ \delta E_{C_2}=-E_{12}+E_{23},$

confirm flip the second qubit

$$\frac{1}{\sqrt{2}}(|000\rangle - |110\rangle)$$

to keep the state $|000\rangle$ still unchange, we need the length of pulse further satisfies the condiditon $\sin(t_3\sqrt{(E_{J_3}/2)^2+E_{23}^2}/\hbar)=0$ by settign E_{J_3} properly.

$E_{J_1}=E_{J_2}=0,\ \cos(E_{J_3}t_3/2\hbar)=0,\ \delta E_{C_3}=-E_{23},$

confirm flip the third qubit

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle - i|111\rangle)$$

The generated GHZ-state is long-lived!, if $n_{g_k} = 1/2, \Phi_{\mathrm{k}} = \Phi_0 / 2$. In fact, for Hamiltoian

$$H_{\mathrm{int}} = -\frac{E_{12}}{2}\sigma_{z_1}\sigma_{z_2} - \frac{E_{23}}{2}\sigma_{z_2}\sigma_{z_3}$$, we have

$$H_{\mathrm{int}} \,|\, \psi_{GHZ} \rangle = (-\frac{E_{12}}{2} - \frac{E_{23}}{2}) \,|\, \psi_{GHZ} \rangle$$

2. Confirm the generated state is just the expected GHZ-state

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + i|111\rangle)$$

Single-shot measuring three boxes simultaneously

*three qubits at the same logic states: This is necessary but **not enough**, as mixture state a|000><000|+b|111><111|*

may also.

$$\tilde{U}_2 = \exp(i\pi\sigma_2^x/4)$$

$$\frac{1}{\sqrt{2}}(|000\rangle - |101\rangle + i|010\rangle + i|111\rangle)$$

$$\hat{P}_2 = |1\rangle\langle 1|$$

$$|0_2\rangle \otimes \frac{1}{\sqrt{2}}(|0_1 0_3\rangle + |1_1 1_3\rangle)$$

$$\prod_{j=1,3} \exp(i\pi\sigma_j^x/4)$$

Quantum interference!

$$\frac{1}{\sqrt{2}}(|0_1 1_3\rangle + |1_1 0_3\rangle)$$

Single-shot measurements: ---- two qubits are not at the same logic states!

How to implement the single-qubit rotations?, e.g.,

$$\tilde{U}_2 = \exp(i\pi\sigma_2^x/4)$$

Let the circuit evolves under the Hamiltonian

$$\hat{H}_2 = -2\epsilon_{J_2}\sigma_2^x + \sum_{j=1}^{2} K_{j,j+1}\sigma_j^z\sigma_{j+1}^z$$

$$\zeta_{j2} = K_{j,2}/2\epsilon_{J_2} < 1$$

Approximately

$$\hat{H}_2^{eff} = -\epsilon_{J_2}[1 + 2\zeta_{12}^2 + 2\zeta_{23}^2 + 4\zeta_{12}\zeta_{23}\sigma_1^z\sigma_3^z]\sigma_2^x$$

In GHZ-state $\sigma_1^z\sigma_3^z = 1$

$$\hat{H}_2^{eff} = -\epsilon_{J_2}[1 + 2\zeta_{12}^2 + 2\zeta_{23}^2 + 4\zeta_{12}\zeta_{23}]\sigma_2^x$$

3. Application: Testing Bell's theorem without inequalities

NATURE | VOL 403 | 3 FEBRUARY 2000

Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement

Jian-Wei Pan*, Dik Bouwmeester†, Matthew Daniell*, Harald Weinfurter‡ & Anton Zeilinger*

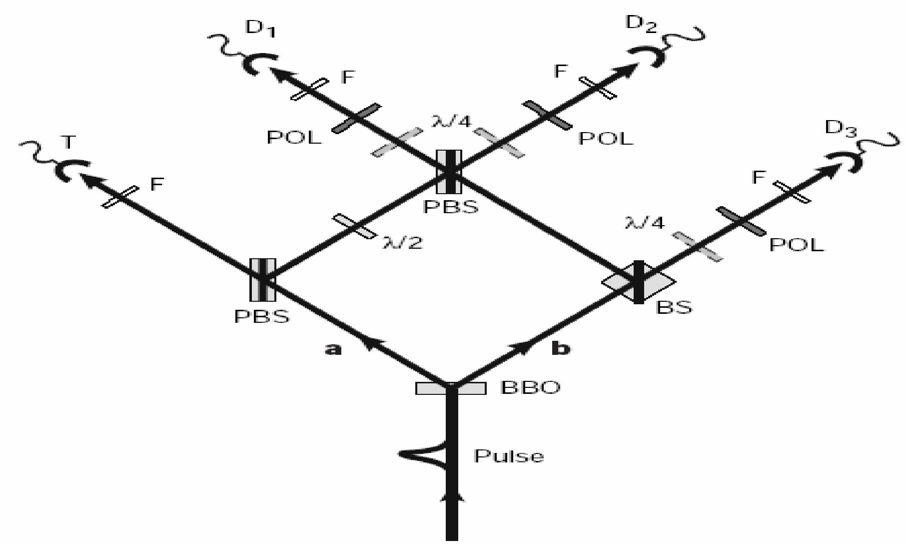


Figure 1 Experimental set-up for Greenberger–Horne–Zeilinger (GHZ) tests of quantum nonlocality. Pairs of polarization-entangled photons[28] (one photon *H* polarized and the other *V*) are generated by a short pulse of ultraviolet light (~ 200 fs, $\lambda = 394$ nm). Observation of the desired GHZ correlations requires fourfold coincidence and therefore two pairs[29]. The photon registered at T is always *H* and thus its partner in **b** must be *V*. The

A proposed experiment with solid-state systems:-principle

It is easily proven that the generated GHZ-state

$$|\psi_{\mathrm{GHZ}}\rangle = \frac{1}{\sqrt{2}}(|000\rangle - i|111\rangle)$$

is an eigenstate of the operators $\sigma_{\mathrm{y}_1}\sigma_{\mathrm{x}_2}\sigma_{\mathrm{x}_3}, \sigma_{\mathrm{x}_1}\sigma_{\mathrm{y}_2}\sigma_{\mathrm{x}_3}, and\ \sigma_{\mathrm{x}_1}\sigma_{\mathrm{x}_2}\sigma_{\mathrm{y}_3}$:

$$\sigma_{\mathrm{y}_1}\sigma_{\mathrm{x}_2}\sigma_{\mathrm{x}_3}|\psi_{\mathrm{GHZ}}\rangle = \sigma_{\mathrm{x}_1}\sigma_{\mathrm{y}_2}\sigma_{\mathrm{x}_3}|\psi_{\mathrm{GHZ}}\rangle = \sigma_{\mathrm{x}_1}\sigma_{\mathrm{y}_2}\sigma_{\mathrm{x}_3}|\psi_{\mathrm{GHZ}}\rangle$$

$$= \frac{1}{\sqrt{2}}[|111\rangle(-i) - i|000\rangle(i)] = |\psi_{\mathrm{GHZ}}\rangle$$

$$\sigma_{\mathrm{y}_1}\sigma_{\mathrm{y}_2}\sigma_{\mathrm{y}_3}|\psi_{\mathrm{GHZ}}\rangle = \frac{1}{\sqrt{2}}(\sigma_{\mathrm{y}_1}\sigma_{\mathrm{y}_2}\sigma_{\mathrm{y}_3}|000\rangle - i\sigma_{\mathrm{y}_1}\sigma_{\mathrm{y}_2}\sigma_{\mathrm{y}_3}|111\rangle)$$

$$= \frac{1}{\sqrt{2}}(|111\rangle(-i)^3 - i(i)^3|000\rangle) = \frac{1}{\sqrt{2}}(+i|111\rangle - |000\rangle)$$

$$= -|\psi_{\mathrm{GHZ}}\rangle$$

In the other hand,

$$(\sigma_{y_1}\sigma_{x_2}\sigma_{x_3})(\sigma_{x_1}\sigma_{y_2}\sigma_{x_3})(\sigma_{x_1}\sigma_{x_2}\sigma_{y_3})$$

$$=(\sigma_{y_1}\sigma_{x_2}\sigma_{x_3})(\sigma_{x_1}\sigma_{x_1}\sigma_{y_2}\sigma_{x_2}\sigma_{x_3}\sigma_{y_3})$$

$$=(\sigma_{y_1}\sigma_{x_2}\sigma_{y_2}\sigma_{x_2}\sigma_{y_3}) = -\sigma_{y_1}\sigma_{y_2}\sigma_{y_3},$$

Thus,

$$(\sigma_{y_1}\sigma_{x_2}\sigma_{x_3})(\sigma_{x_1}\sigma_{y_2}\sigma_{x_3})(\sigma_{x_1}\sigma_{x_2}\sigma_{y_3})|\psi_{\mathrm{GHZ}}\rangle = |\psi_{\mathrm{GHZ}}\rangle,$$

results in the following quantum equality

$$\sigma_{y_1}\sigma_{y_2}\sigma_{y_3}|\psi_{\mathrm{GHZ}}\rangle = -|\psi_{\mathrm{GHZ}}\rangle$$

In the local world, each operator corresponds to a classical quantities:

Quantum: $\sigma_{\mathrm{x,y,z}} \rightarrow$ classical: $m_{x,y,z}$ =+1 or -1; $m_{x,y,z}^2 = 1$

Quantum: $\sigma_{\mathrm{y}_1}\sigma_{\mathrm{x}_2}\sigma_{\mathrm{x}_3}|\psi_{\mathrm{GHZ}}\rangle = \sigma_{\mathrm{x}_1}\sigma_{\mathrm{y}_2}\sigma_{\mathrm{x}_3}|\psi_{\mathrm{GHZ}}\rangle = \sigma_{\mathrm{x}_1}\sigma_{\mathrm{y}_2}\sigma_{\mathrm{x}_3}|\psi_{\mathrm{GHZ}}\rangle = |\psi_{\mathrm{GHZ}}\rangle$

$\Downarrow$

Classical: $m_{y_1} m_{x_2} m_{x_3} = 1, m_{x_1} m_{y_2} m_{x_3} = 1, m_{x_1} m_{x_2} m_{y_3} = 1$

$\Downarrow$

Classical locality: $m_{y_1} m_{x_2} m_{x_3} m_{x_1} m_{y_2} m_{x_3} m_{x_1} m_{x_2} m_{y_3} = 1$

$\Rightarrow m_{y_1} m_{y_2} m_{y_3} m_{x_1}^2 m_{x_2}^2 m_{x_3}^2 = 1$, resulting in the classical locality

$$m_{y_1} m_{y_2} m_{y_3} = 1,$$

which is in contradiction with the quantum prediction:

$\sigma_{\mathrm{y}_1}\sigma_{\mathrm{y}_2}\sigma_{\mathrm{y}_3}|\psi_{\mathrm{GHZ}}\rangle = -|\psi_{\mathrm{GHZ}}\rangle \Rightarrow m_{y_1} m_{y_2} m_{y_3} = -1!$

So, experimentally,

i) Generating the expected GHZ state and measure $\sigma_{y_1}\sigma_{x_2}\sigma_{x_3}$

ii) Generating the expected GHZ state and measure $\sigma_{x_1}\sigma_{y_2}\sigma_{x_3}$

iii) Generating the expected GHZ state and measure $\sigma_{x_1}\sigma_{x_2}\sigma_{y_3}$

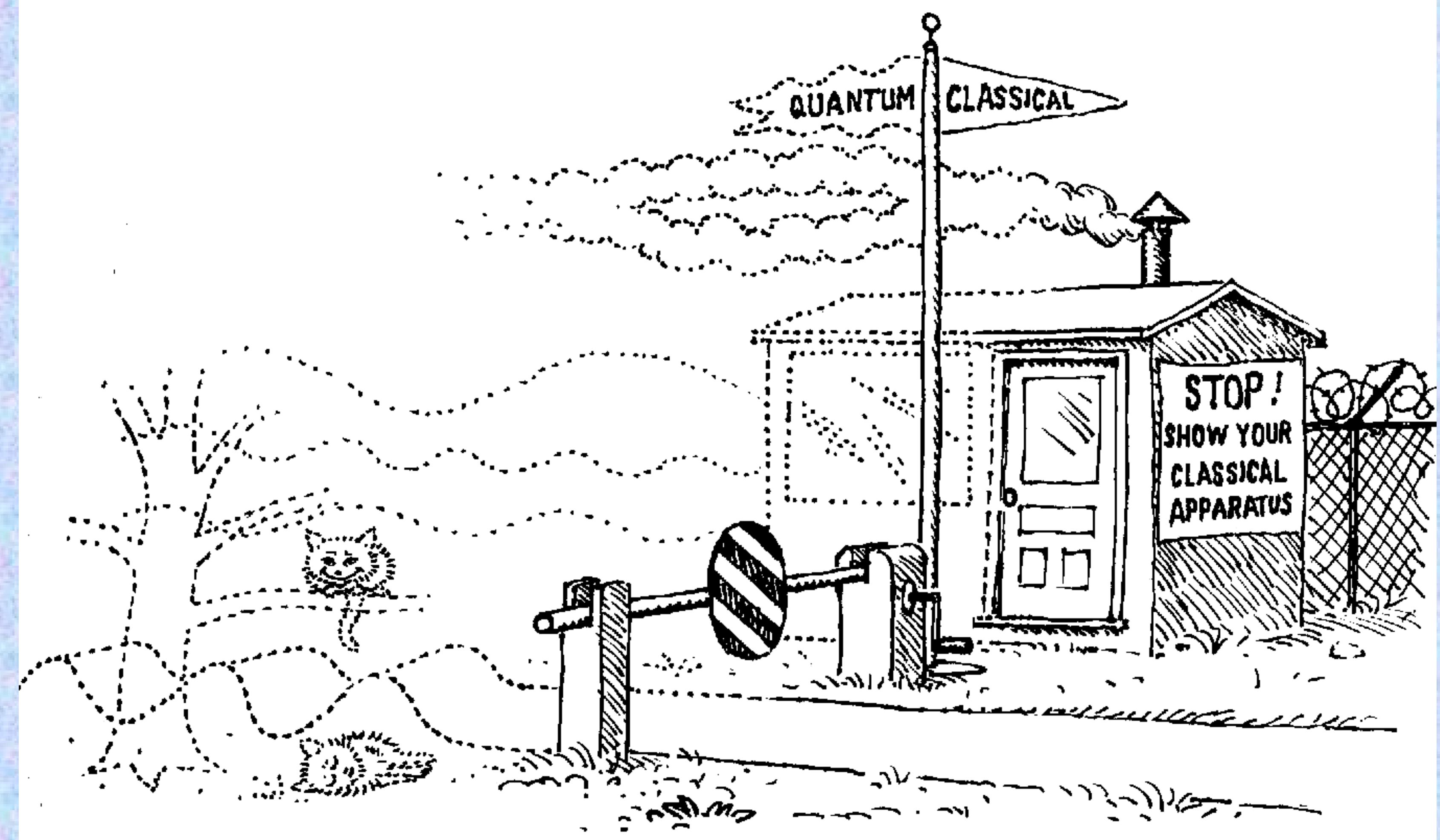
Calculating $m_{y_1} m_{x_2} m_{x_3} m_{x_1} m_{y_2} m_{x_3} m_{x_1} m_{x_2} m_{y_3} = ?$

iv) Generating the expected GHZ state
and perform ***the fourth*** tested experiment: measuring $\sigma_{y_1}\sigma_{y_2}\sigma_{y_3}$

Quantum result $m_{y_1} m_{y_2} m_{y_3} = -1$ conflicts the classical one $m_{y_1} m_{y_2} m_{y_3} = 1$

- **Fundamental physical motivation:**

  Entanglement lies at the heart of quantum mechanics.



Superconducting circuit to test quantum theories

Device could form basis of powerful quantum computer

A simple circuit that tests the boundaries of quantum physics has been devised by RIKEN scientists. The proposed device could also be a key element of a future quantum computer, where information is stored and processed using the quantum properties of sub-atomic particles.

The circuit relies on three tiny superconducting devices that can carry electrical current, in the form of pairs of electrons, with virtually no resistance (Fig. 1). If trapped within the superconductor, odd or even numbers of electron pairs can represent the opposing states of binary logic that match the stream of 'ones' and 'zeros' used in conventional computers.

The data can be retrieved by counting the total number of electron pairs held in each superconducting 'box'.

But there's a catch. Although each box contains billions of electron pairs, making it far from sub-atomic, the behavior of the electrons is governed by the rules of the quantum world. This makes it possible for the overall quantum state of each box to become entangled with its neighbors so that they share the same information, as if they were part of a unified whole. Einstein himself dubbed this strange entanglement as 'spooky action at a distance', and it means that changing—or even measuring—the status of one of the superconducting electron-pair boxes can instantaneously affect the other two.

So each box essentially behaves as a unique quantum particle, and linking the three in this way is known as Greenberger-Horne-Zeilinger entanglement, named after the scientists who first described it in 1989.

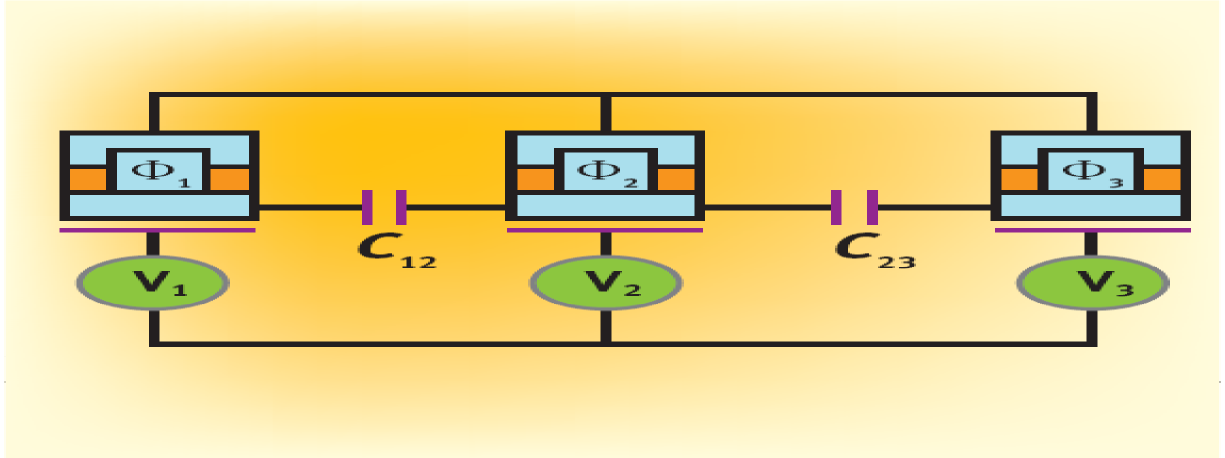


Figure 1: The three superconducting devices linked together in this circuit could form a key element of a future quantum computer.

"We propose that these phenomena can also be observed in the macroscopic world, using circuits, and not only in the microscopic realm," explains L. F. Wei of RIKEN's Frontier Research System in Wako, who devised the circuit with his colleagues Yu-xi Liu and Franco Nori, who is also at the University of Michigan, USA. Their proposal is published in *Physical Review Letters*[1].

The three superconducting boxes should be much easier to manipulate into specific quantum states than individual electrons, and controlling these 'macroscopic' quantum states provides a way to test some fundamental laws of quantum mechanics, such as entanglement, at the macroscopic level, says Nori. "But it is also a key step to building future quantum computers," he adds.

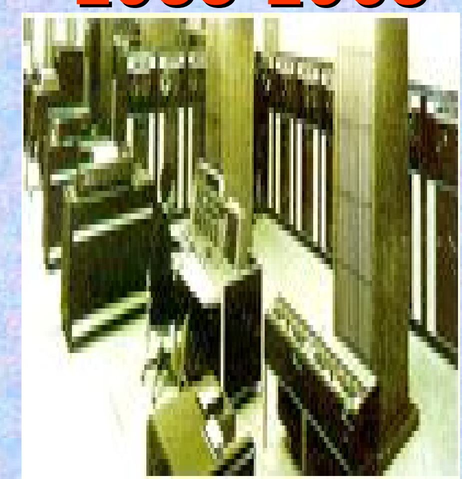
Since quantum states can exist in various combinations known as superpositions, there are many more configurations available to the three superconducting boxes than if they were a simple series of classical computer bits, which substantially increases the system's computing power. "We believe that the proposed system could be experimentally built in the near future," says Wei.

1. Wei, L. F., Liu, Y-X. & Nori, F. Generation and control of Greenberger-Horne-Zeilinger entanglement in superconducting circuits. *Physical Review Letters* 96, 246803 (2006).

Towards to Quantum Computer

1945-1955

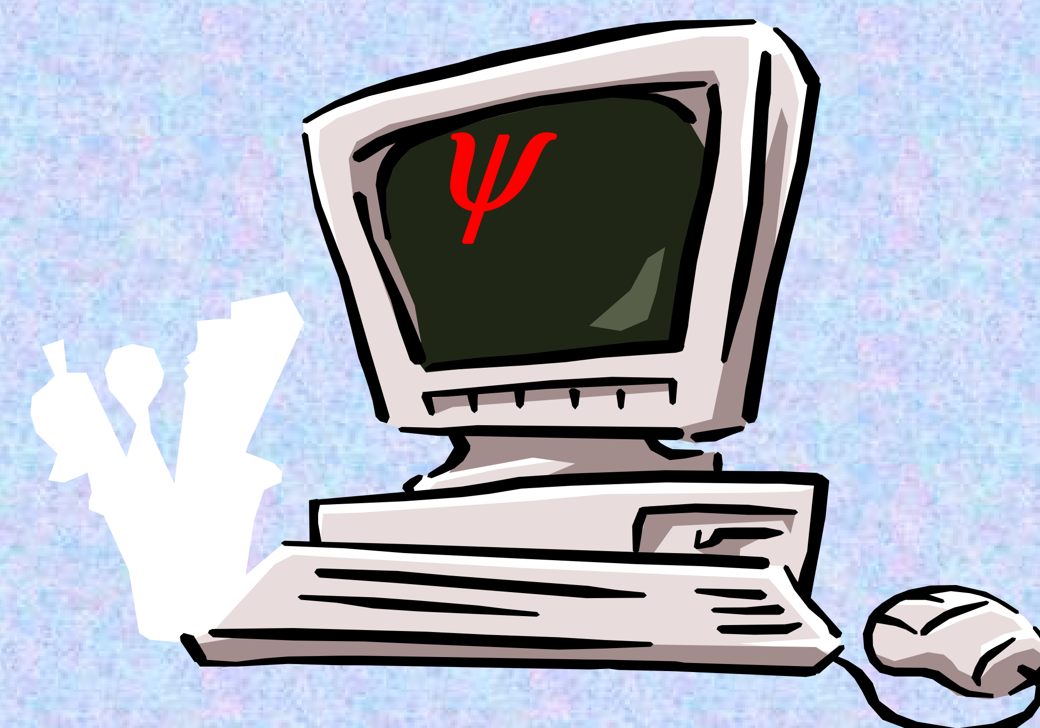
1955-1965



集成电路

1985-2008

Quantum Computer?



问君能有几多愁

春花秋月何时了，

往事知多少！

小楼昨夜又东风，

故国不堪回首月明中。

雕栏玉砌应犹在，

只是朱颜改。

问君能有几多愁？

恰似一江春水向东流！

Thank you